**Rabi Cycle**

Now let’s consider a two-state system.



I guess we won’t specify H0 per se´, but will presume that the state |χ(t)> starts out in an eigenstate (either |1> or |2>) of H0, and that it’s energies are E1,2. Actually, we will make an interpretation. Let’s suppose we’re dealing with an electron in a Hydrogen atom, say. So H0 is the Hydrogen atom Hamiltonian, but we’ll only be interested in two states in its Hilbert Space. We’ll say it starts in the ground state |χ1> = |n=0,ℓ=0,mℓ=0>, and we’ll be interested in transitions to the first excited state |χ2> = |n=1,ℓ=1,mℓ=0>, say. Further, we’ll presume that these are the only two states it can transition between. The transitions will be spurred by = E∙. So let’s consider a general state:



and work out equations for the coefficients.



We’ll dot both sides with <1| and <2|,



Or for short,



We’ll go to the interaction picture, kind of. Let’s define,



and plug these in,



And this simplifies to:



and simplifies a bit more, to,



Now we’ll note that, due to parity at least, the matrix elements V11 and V22 must be zero. And let’s call ω12 = (E1 – E2)/ℏ. So then we have:



A common approximation at this point is to express cos(ωt) in the complex exponential form and disregard the ω+ω12 term as too fast to matter to the behavior. Basically, only the ω – ω12 term matter as ω 🡪 ω12. This is called the rotating wave approximation. So,



and,



The detuning frequency is defined as:



and in terms of this, we may write,



Don’t know how to proceed, except like this. I’ll multiply top by eiδt, differentiate, and then plug in the bottom term,



This is a linear first order ODE. Let’s do the usual trial solution 1(t) = e-iat.



So,



Let’s specialize to 1(0) = 1. And 2(0) = 0. Then clearly A = 1.



And then use the equation:



to infer,



So then we have:



And going back to:



we can say,



So then,



This result shows that the amplitude (square root of probability) of transition oscillates with amplitude and frequency:



The amplitude is highest, and frequency smallest, when δ = 0 🡪 ω = ω12, the energy level difference. We’ll also note the amplitude is proportional the strength of the perturbation, and zero when the perturbation vanishes.