**2nd quantization**

Guess we’ll try out a few bases…

**Some operators in the momentum basis**

We start by constructing the density operator in the momentum basis, which is fairly easily done in your head by keeping in mind the Fourier transform facts. To remember the fact that the momentum gets lowered, recall that *densities* *demote* and *potentials promote*. And *V(x)* has a factor of *volume* in it, while *density* *doesn’t*.



and the Fourier transform would give us:



which can be written as:



Now consider the FT of the current density:



Trying to use those relations way up above, and Fourier transform of product rules (see Fourier Transform file)…



which comes to:



(emphasizing the vector-ness of k, q, and note how k - q/2 is the average of the momenta in the indices and when divided by m, gives the average velocity). Let’s do momentum,



and so we have:



In the same way, we can get the kinetic energy representation:



The single particle *spin-independent* potential, V1, can be represented in momentum space, via (could do this direct from the k-basis, but I’m just going to convert the position basis formula to the k-basis one)



So,



If we presumed spin-independent potential: V1σσ´(q) = V1(q)δσσ´.



To recall that we have momentum creation modulated by the single particle potential, recall that *potentials promote*. And finally we have the representation for the two particle potential, V2 (presumed spin-independent)



and so we have:



The formula is generally considered to state that the inner pair of operators go together and the outer pair go together so that the state goes to the state ; and the state goes to the state . There are many ways to write this last formula; the only important thing is that the sum of the momenta of the initial states (annihilation operators) is equal to the sum of the momenta in the final state (creation operators). And also that the inner spins are equal, and the outer spins are equal. Because there is no spin dependent part of the potential, the spins of the individual particles are conserved. One often times writes the second quantized two particle interaction operator as:



where



but this last form is somewhat defective because the operators aren’t in the correct order. And the present form has a term where the particle interacts with itself – I suppose he means a number operator or something. So for instance, in order to put V2 in the last form, we’d have to do,



So perhaps the last term could simply be subsumed into the chemical potential. A potential problem with even the first formulation, from a practical point of view, is that the Fourier transform may not exist. But this problem can always be avoided by reformulating the problem in terms of the T-matrix, which always has a Fourier transform – so they say.

**Time development in momentum space**

Assuming a free particle Hamiltonian



Then we have



It develops harmonically, which is indicative of the fact that ck is an annihilation operator, just like we established before in QM1. Do not lose sight of the fact that examining the time development of these operators can prove just as useful for determining eigenstates/eigenvalues as examining the time development of the other operators was. Maybe let’s do the full thing. So we’ll presume a Hamiltonian of the sort,



and we’ll recall with a generic H given by



the time-development of a was:



So we have:



Let’s work out the matrix element,



So,



So,



**Translation Operator**

So I’d like to consider what happens when we operate on the creation/annihilation operators with various symmetry operators. So I’ll do the translation operator first,



So operating on it, we have:



where the last line is using the Baker-Hausdorf formula (see Foundations/Continuous Symmetries). So we have to evaluate that commutator,



So we can see that:



which is interesting – as I just assumed the phase would be zero. But okay. Now we can take the dagger of both sides as well, and together we’ll have:



Let’s verify that the two particle potential is translationally invariant,



So it checks out.

**Spin-Rotation Operator**

We can represent the spin-rotation operator as:



Let’s say we did a 180o spin-rotation about the y-axis. Then using the rotation operator D(π**j**) = e-iπS\_y, and recalling from the QM Foundations/Rotation file that: D(π**j**)|σ> = -sgn(σ)|σ>, we have:



And taking the dagger, we’d get:



Multiplying both sides by D†(α)….D(α), we get:



And we can do rotations about the x-axis too, like we explored in the position space case. We should find:



**Parity Operator**

Let’s do the parity operator.



and taking the dagger of both sides, we have altogether,



Is the two particle potential symmetric under parity?



So it is as well.

**Time-Reversal Operator**

Now let’s look at the Time Reversal Operator.



I wonder about that phase factor, but I’ll keep it. Taking dagger of both sides might be kind of problematic. So I’ll just repeat:



Surrounding both sides of our equation with Θ-1….Θ (and recalling that Θ is anti-linear), we would now see that:



But of course this is wrong (see position space note), and we should have instead:



which could be justified within our derivation if we say Θ|0> = i|0>, but who knows.