**Douglas Expansion of N-particle Propagator**

Kind of have to look ahead for GF stuff to make sense of this.

**Introduction**

In the previous chapter the GDMPK equation was reduced to a complex time Schrödinger equation. Let us consider this equation in generality. So we suppose we have N particles in a potential field, subject to single particle boundary conditions, and mutually interacting. We will consider the function G(**x**,t|**y**) = G(x1,…,xN;t|y1,…,yN), which will satisfy the Schrödinger equation in the variable x.



and the coordinates will be assumed to range between *a* and *b*. We will assume δ function initial conditions, from which any other desired initial condition can be easily obtained. We’ll be interested in two cases – symmetric initial conditions, and anti-symmetric initial conditions.



where π(**y**) signifies all permutations of the order of the coordinates y1, y2, …yN, and gives 1 for all permutations of **y** if we’re interested in the symmetric case, or gives 1 for even permutations of **y** and -1 for odd permutations if we’re interested in the anti-symmetric case. We will also consider that it obeys single particle boundary conditions of the form,



for each coordinate xj. These conditions may take the form of enforcing periodicity in each coordinate, or that the Green’s function vanishes at the endpoints of each coordinate’s interval, etc.

**Expression for GN in 1st Quantized Notation**

Let |j> enumerate single particle states that satisfy the boundary conditions. Then let,



enumerate symmetrized (or anti-symmetrized) N-particle states constructed out of these. And let,



be symmetrized (or anti-symmetrized) N-particle position eigenkets. Then the solution to the Schrödinger equation is merely the N-particle propagator in position space, expanded in the basis that satisfies the single particle boundary conditions.



We see that this equation does satisfy the Schrödinger equation, for we have,



This expression also satisfies the initial conditions as can be seen by setting t = 0, and evaluating the result using the definition of |**x**> and |**y**>. The boundary conditions are satisfied as well, as long as one uses the representation above in terms of the complete basis of single particle states which satisfy those boundary conditions. This formulation of the solution lends itself perhaps to path-integral methods of calculation, in addition to perturbative Feynman diagram expansions. We’ll opt for the latter. The first quantized form of GN us to segue into a second quantized formulation.

**Expression for GN in Second Quantized Notation**

In order to develop a Feynman diagram expansion of this expression, we’ll like to go to second quantized notation. Let |**0**> denote the N-particle vacuum state. And let ψ†(j) create a particle in the state |j> which again is a single particle state satisfying the boundary conditions. If we desire symmetric initial conditions then we’ll demand the ψ’s obey bosonic statistics. If we want anti-symmetric initial conditions, then ψ will satisfy fermionic statistics. Then we can write G(**x**;t|**y**) as:



In the second line we use the fact that H, given below, annihilates the vacuum:



This expression for G satisfies the partial differential equation, as can be demonstrated by explicitly evaluating its time-derivative, and using the partial differential equation satisfied by the position space annihilation operator.

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where,



and



It can be verified that the initial conditions are satisfied by setting t = 0 and moving all of the ψ’s to the right of the ψ†’s using the commutation relations on ψ, ψ†. And finally, the boundary conditions are satisfied if we use the eigenbasis which satisfies the boundary conditions to expand the position-space field operators, ψ, ψ†. As is typical, it is advantageous to switch to the interaction picture. So we introduce,



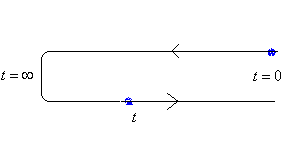
Since S also annihilates the vacuum, we can insert S and S† beside the ψ’s in a propitious manner to come to the following expression,



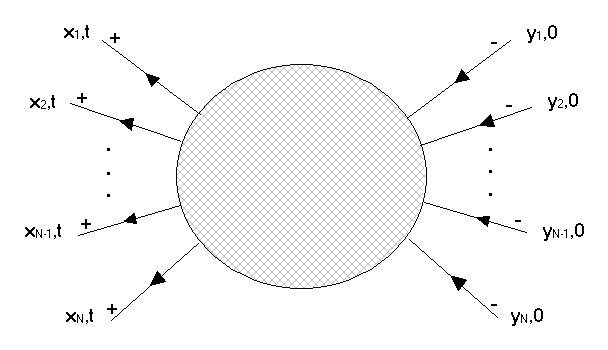
which we can notate as,



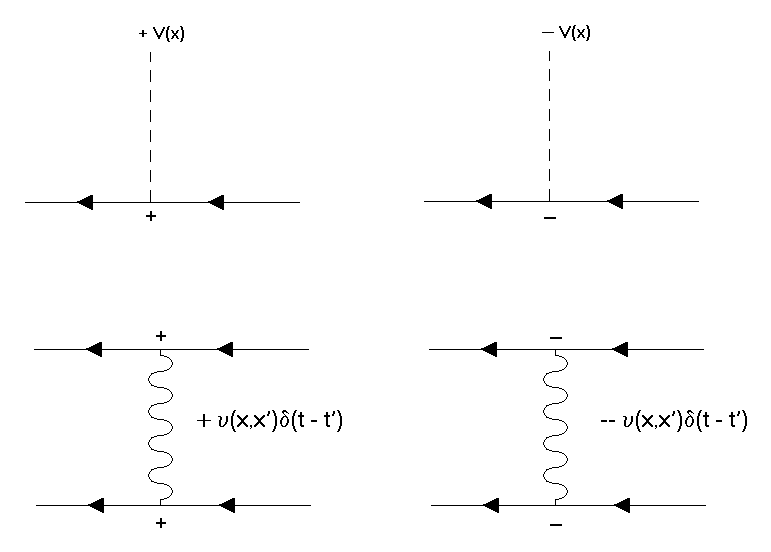
where SC is the time evolution operator which contour orders the operators from t = 0, to t = ∞, and then back to t = 0. Instead of evolving from 0 to infinity and back, we may evolve from 0 to any finite time and back, but the calculations simplify if we extend the time evolution to infinity. Our evolution contour is illustrated below.



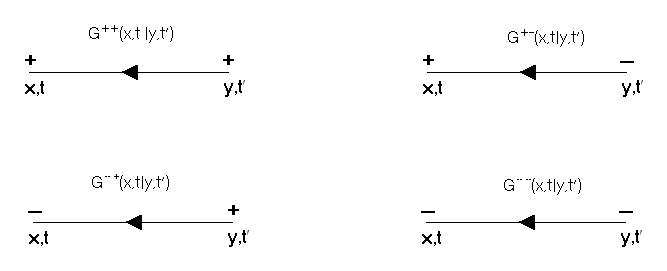
These contour ordered Green’s functions are discussed in the appendix. The perturbative calculation of this quantity follows from standard perturbation theory [18]. We assign N incoming propagators at the coordinates xm, t, and N outgoing propagators at the coordinates yn, 0.



And we connect these propagators together via the interaction vertices:



These diagrams will then involve four different Green’s functions.



And they are given below. Assuming that the eigenfunctions (satisfying the boundary conditions) of the unperturbed Hamiltonian are φk­(x), the 4 Green’s functions would explicitly be:



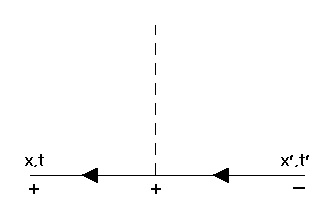
And we would integrate over all internal positions from a to b, and all internal times from 0 to ∞. We’ll note that the G> isn’t defined for (because the sum diverges), unlike in the real time case. So G> is only partially defined. This same issue makes G++ completely undefined. G— is the only well defined green’s function. Nonetheless, we will formally work with these functions, and find that in the end completely finite results will emerge, completely analogous to the results we would find in the real time case.

**General Features of the Diagrammatic Expansion**

The diagrammatic expansion delineated above simplifies considerably. And we will consider these consequences below. First we will find that the expansion reduces to that of the time-ordered Green’s function for all positive ‘times’. Secondly, we will find that a large class of diagrams typically associated with such diagrammatic expansions do not appear. That is, there will be no self-energy diagrams, no vertex corrections, and no crossed diagrams. Self energy diagrams will be absent because the self-energy is the interaction of the particle with the background field. But our background field is the vacuum. In a similar vein, the interaction itself will not experience any corrections because there will be no background to mediate it. Crossed diagrams can be thought of as propagation backwards in time, but this will be disallowed as we will see. All propagators must proceed forwards in time.

**Equivalence to the Time-Ordered Green’s Function**

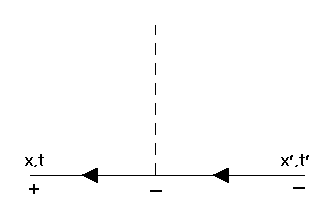
When we work out the diagrammatic expansion of G> we will find that the infinities inherent in the expression cancel, and the finite parts reduce to the expansion of the time-ordered Green’s function. To illustrate, we’ll look at the first order diagrams in the single particle potential and interaction. To first order in the external potential, there are only two diagrams. Let’s consider one of them.



This diagram is given by the expression,



Its sister diagram is the following:



which is given by,



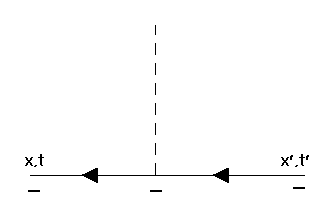
This diagram is infinite too. But adding them together gives,



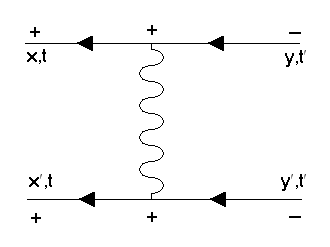
which is finite. So we’ll note that the two structurally identical diagrams add to give one result which can be written as



i.e,



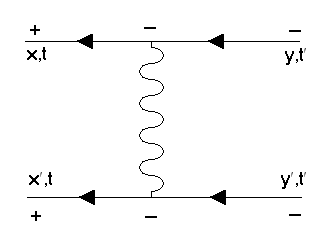
Let us consider the first order term in the interaction potential. Again there are two contributing diagrams. One is:



Without loss of generality we’ll set . Then we’ll have,



And its sister diagram:



is given by,



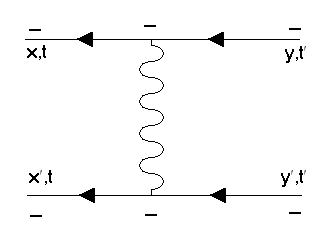
Each is infinite, but adding them together gives



which is finite. Again we’ll note that the two structurally identical diagrams add to give one result which can be written as



which is,



It has been verified out to second order that this reduction continues. Therefore it seems plausible that the a priori divergent calculation of G> reduces to the completely finite calculation of G--. This is not surprising since we always consider positive times (since time is a length) and for t > 0,



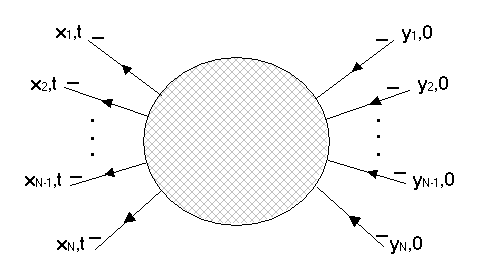
is identical to:



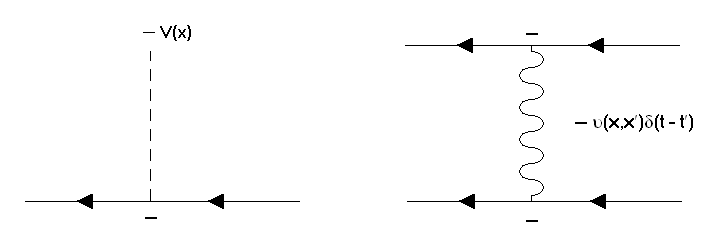
Therefore the diagrammatic expansions should be equivalent. But we will note that there is an implicit θ(t) function in the definition of G—(**x**,t|**y**,0). Therefore it will satisfy a trivially different PDE,



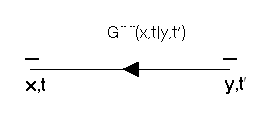
which for times greater than 0, reduces to what we are interested in. Therefore, we may replace our rules with the following. Calculate all diagrams of the form,



And we connect these propagators together via the interaction vertices:



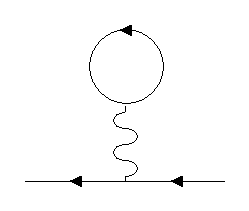
which involves only the single green’s function,



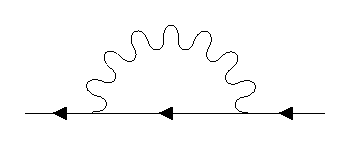
And we would integrate over all internal positions, and times from 0 to ∞. From the theta function in the definition of G-- we see that in the diagrammatic expansion, particles only propagate forwards in time. The arrow points towards later times. As a general rule, any diagram which would force the particle to propagate backwards in time will be 0. This eliminates a large class of diagrams from the expansion. For the following we’ll neglect any single particle potential present in the Hamiltonian. Its presence does not affect the substance of the following observations.

**Absence of Self-Energy Diagrams**

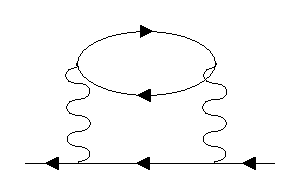
We will generally find that the bare propagator receives no self energy corrections. Consider the Hartree self-energy term. From now on, we’ll leave off the extraneous (-) signs on the vertices.



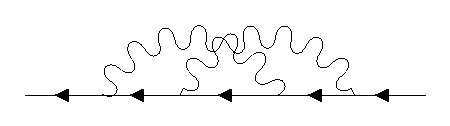
The loop propagator is interpreted as the expectation of the particle density. But this is 0 since we are taking the expectation with respect to the vacuum. Next, consider the Fock diagram.



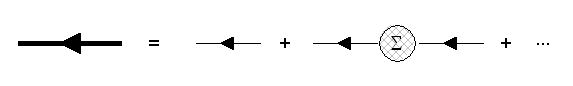
This diagram is 0 for the same reason. The δ function in the potential line requires the middle propagator to be interpreted as the particle density. Next let us consider the three second order Σ diagrams.



This diagram is 0 because the propagator at the top of the bubble requires later times to proceed from left to right, while the one at the bottom of the bubble requires later times to proceed from right to left. Therefore, the two propagators cannot be both be non-zero at the same time. Another is:



This is also 0 because the right side interaction line operates at time t1, and the left side interaction line operates at time t2. If t1 > t2, then the second from right propagator is 0. If t1 < t2, then the middle propagator is 0. The other second order Σ diagram (Fig. 2-17) is 0 for the same reason the Fock diagram is 0. A general argument for the absence of any self energy term can be made. Consider the N = 1 case. This Green’s function, G1, would have a general diagrammatic expansion of the form,



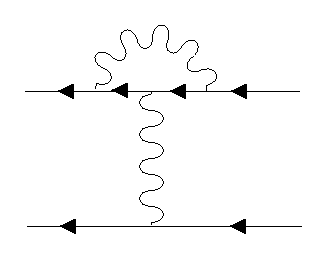
But the differential equation satisfied by G1 is:



which is the same as that satisfied by the bare propagator. Thus G1 is simply the bare propagator. And therefore there can be no self energy corrections.

**Absence of Vertex Corrections**

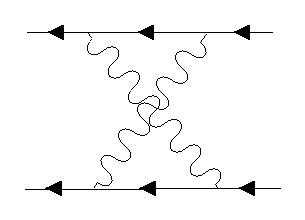
Consider the following proto-typical vertex correction.



The upper interaction line operates at t­1 and the lower one operators at t­2, say. Again, if t1 > t2, then the propagator second from right is 0. If t­1 < t2, then the propagator second from left is 0. Making similar arguments, we should find that the interaction receives no corrections in general.

**Absence of Crossed Diagrams**

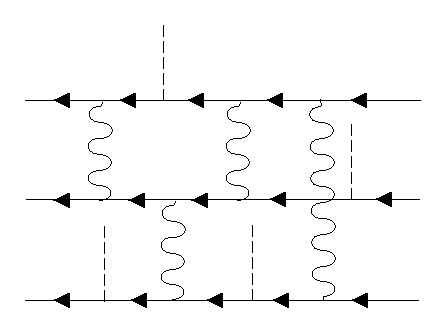
Consider a typical crossed diagram.



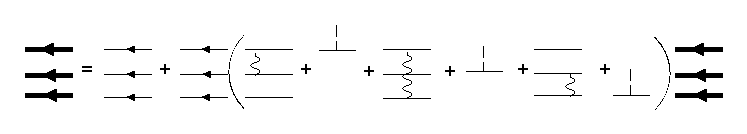
This diagram too is 0. Let the back-slash interaction line be t1 and the forward slash interaction line be t2. Depending on which time is greater, either the middle top or middle bottom propagator will be 0.

**General Form of the Diagrammatic Expansion**

Given the arguments above, we can observe that the general form of a term in the diagrammatic expansion will be like the following, illustrated for N = 3. We have N horizontal lines for N particles. And all the interactions take place along the vertical, connecting any two particle lines. External potential lines can be inserted anywhere.



We can prove this supposition by the following argument. Taking N = 3 for illustrative purposes again (the argument for general N is completely analogous), supposing the above is correct, G3 can be constructed via the following recursive equation,



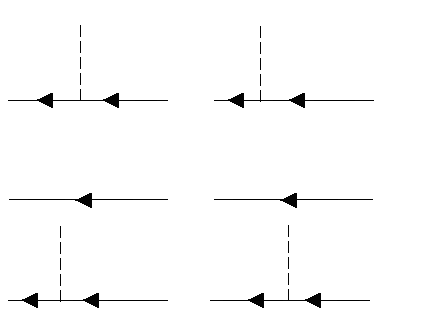
This would be written in 4-vector notation as:



If we operate on this equation with the single particle differential operator, -∂/∂t – H0, (where H0 includes the single particle potential), then we obtain,



which is indeed our differential equation, for times greater than 0. Therefore this is the correct diagrammatic expansion. Note that this implies that diagrams which we normally think of as identical are in fact distinct. For instance,



are distinct diagrams, because the interactions occur at different times. Each interaction (external or internal) will occur at a specific time, which proceeds to the right. And if interactions occur at different times, then they are distinct.

I don’t think this is correct. Note that the expansion of a two-particle GF with just V(x)’s, and no two-particle interaction is just the product of two single particle GF’s. And that expansion isn’t quite what follows from the conceit above.



**Extension of Temporal Integration to the Entire Real Line**

Instead of integrating over the internal times from 0 to ∞, we may without error integrate between -∞ and ∞, because the Green’s functions connecting to the internal time points on the vertices are 0 for any negative time. This extension puts the diagrams in a temporally translationally invariant form so that we may use examine the diagrams in frequency space.

**Prelude on Exponential Expansions of Green’s Functions**

An aside from Mahan, on exponential resummations in general… When correlations are important, an exponential resummation is a possible way to include them. Usually, it works best when describing a few particles interacting against a background. He says there are two possible difficulties with this procedure. The first is that the regrouping may not converge. Nonsensical results are found in cases where G(p,t) describes a GF of a particle in a system with N like particles, where N ∝ V. In that case the Fn don’t converge. The offending terms are those which contribute to the total energy of the system and hence change the chemical potential. These terms are O(V). The chemical potential isn’t changed by V in one-particle or one-hole problems, since the one particle does not contribute any energy term proportional to V. The series shows excellent convergence in this case. I think that he’s saying therefore – if your GS is the Fermi Sea, this generally is a bad approximation, but if your GS is the Vacuum, then maybe not so bad!

The physical model implied by this regrouping will be discussed in Chapter 7. It is a systematic development of the Tomonaga model of pion emission in particle theory. Tomonaga assumed that all pion emissions from a particle were statistically independent – there was no correlation between successive emissions. His model corresponds to the approximation of keeping only F1 in the series. However, if one uses F1 + F2, then F2 puts in the correlation between pairs of emissions. Similarly, F3 puts in correlations between three particle events. The advantage of the method is that the Green’s function includes many-particle emission processes, even when one keeps only a few terms in the series for Fn. Solid state problems do not have pions but other boson excitations such as phonons or the boson-like excitations of the electron gas.

**Exponential Expansion of Perturbative Series (Dissertation)**

Consider the Taylor expansion of the GF, where (ab) stands for the powers of the two perturbations.



where λV refers to the coefficient in front of the single particle potential, and λU the coefficient in front of the interaction. Now let’s consider an exponential series.



F(ab)­ is the abth order term in the exponential series. We can write a few of these Fab in terms of *w*ab = Wab/GN(0). Start with,



Taking ln of both sides and expanding in orders:



Continuing,



equating,

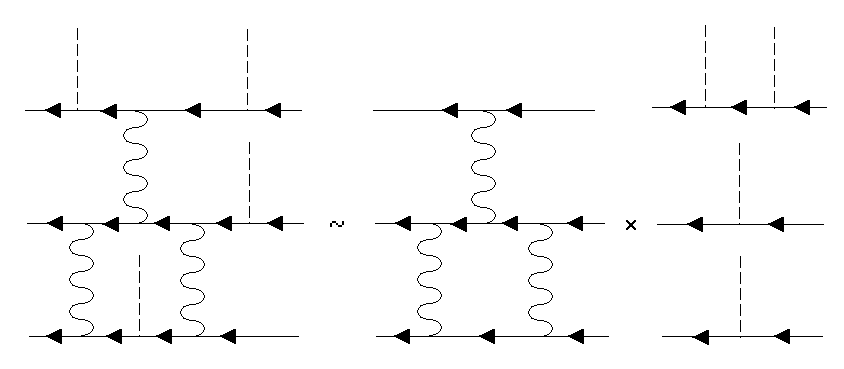


So I’d suggest general rules.



**Separating GN into product of GNV and GNU and remainder**

Now we’ll note that even using the almost exact single particle basis, has not had a significant effect on the two particle interaction term vs. what the free single particle basis produced. Therefore it would seem that the single particle potential contribution to G is somewhat independent from the two particle contribution. We may suppose that this is because the single particle potential does not distinguish between positions (except for very close to the origin) and so it does not influence the expectation of the interaction energy, except close to the origin. Therefore we ought to be able to separate the effects of the single particle potential from the effects of the interaction – especially in the insulating state where only large values for *xn* are relevant. This will also have the advantage of separating the terms which are responsible for satisfying the symmetry requirements of G, from the remainder. We have in mind the following kind of approximation of the diagrams,



where we decouple the external potential and interaction terms. We can do this formally. Let’s go back to the GF expansion and separate out the terms in the exponential expansion which don’t couple V (the single particle potential) and U (the two particle interaction). These are the Fa0 and F0b terms. Then we could write the exponential expansion as,



Now use the fact that by definition,



where GNV(t) is the exact N-particle Green’s functions, exclusive of the interaction, and GNU(t) is the exact N-particle Green’s function, exclusive of the single particle potential. This allows us to write,



Now,



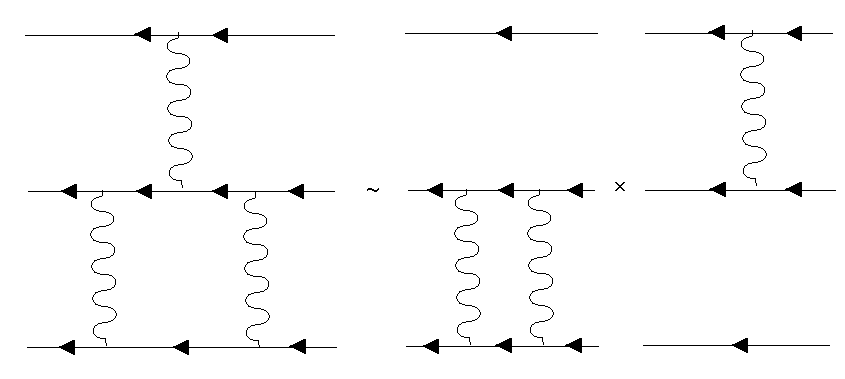
is what we get when we sum *all* diagrams under the diagrammatic approximation illustrated above, while exp[ ] contains all of the corrections to this approximation. Note GNV(t) is exactly determinable, since we can solve for the GF in the single particle case, by exactly solving for the single particle eigenfunctions.

**Separating GNU into Products of G2U and remainder**

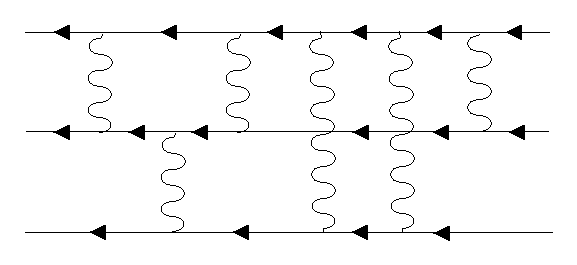
The next question is how we may calculate the N-particle GNU(t)? Whichever approximation we use, we would like to reproduce the known small **y** form of GU, namely that,



so as to preserve the mathematical symmetry of the exact solution. In the same way as above, we will approximate the diagrams in such a way as to separate the singularity canceling behavior from the rest of the behavior. The singularity canceling part we can determine exactly, and the rest we will treat as corrections. What we have in mind is something of the sort,



To that end, instead of the usual perturbative grouping of diagrams, where we group them according to the number of times we bring down the vertex from the time development operator (i.e., the total number of interactions), let us instead group them according to how many times each particle interacts with another. It seems advantageous to do this because as we see above in the symmetry requirement, 2 particle correlations dominate over 3 particle correlations in some sense. Now each diagram will have some number, n, of interactions between particles a and b, where both a and b can run between 1 through N – assuming N particles. For instance, the diagram below,



has 3 interactions between particles 1 and 2, 2 interactions between particles 1 and 3, and 1 interactions between particles 2 and 3. Let j label all possible inter-particle interactions, 12, 13, 14, …, 23, 24, 25, …, 34, 35, 36, …, etc., and let njlabel the number of such interactions. *j* will run between 1 and N!/2 therefore, and nj can take on any value. We’ll label all diagrams therefore with the following notation:



and call



the order of the diagram. For example, the diagram above would be labeled, W321 since there are 3 interactions between particle pair (12), 2 between pair (13), and 1 between pair (23) respectively. And this diagram would be of order λ{321} = λ13λ22λ31. In this notation, the usual Taylor series expansion of the GF would look like,



We would like to reorganize the series according to:



We can guess general rules for the exponential expansion – i.e., general rules for constructing the F’s. We’ll focus on the case where we’re expanding in the exact single particle basis – so there are no external interacting lines. Let’s match the exponential expansion and the usual Taylor series expansion term by term – up to third order. First, the Taylor series expansion would be (for N = 3). And…all these equations are pictures and I wrote F instead of f, but can’t change it now…



let’s take the ln of both sides,



and expand the ln,



Now equating,



The general rules that can be discerned are…again that should be f, not F, but it’s a picture.



Again, we can separate the sum into diagrams which involve only the same particles and those which cross particles. So similar to before, we can write,



where the {nj′} refers to sets that mix particle interactions, for example the term *f*101 above. Now we recognize that analogous to before,



is the two particle Green’s function connecting the nj’th pair of particles. So we may write,



So now using a more conventional notation where G2U(a,b) is the exact 2-particle Green’s function between particles a and b, we may write out the perturbative expansion for GNU to be,



Now



is what we would have if we summed all interaction diagrams under the approximation illustrated in the figure above. And the exp{ } term contains all of the corrections to this approximation.

**Final Result**

So altogether, we would have,



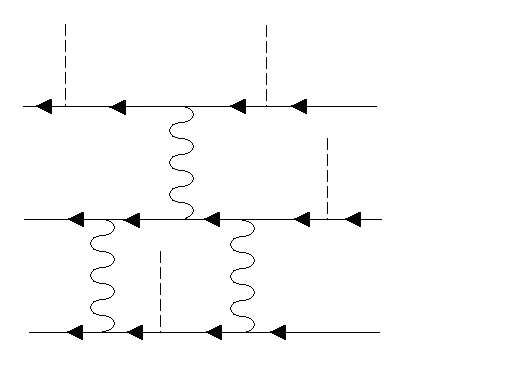
We can approximate GN by leaving off the exponential corrections. Since G1­ and G2 must have the same singular behavior as GN, at a reduced dimensionality of course, we can clearly see that this approximation possesses the requisite singular behavior of GN. We should therefore, in the interests of preserving the mathematical symmetry of the actual solution, take this as our true 0th order approximation, and it would seem that this is the natural way to perturbatively calculate the N-particle propagator. The zeroth order approximation is already quite accurate as we’ll see. And we’ll also note its utility, as it only requires calculating 1-particle, and 2-particle Green’s functions, which can often be done exactly. Additionally, it doesn’t require that the strength of the interaction, or external potential be small, only that two-particle correlations predominate. In principle therefore, this expression could be an adequate starting point for the analysis of the insulating and critical states – all the way up to the metallic state, provided the parameters K11 and K12 are suitably modified. Finally this approximation is well controlled so that corrections can be straightforwardly included. Similar mean field approximations are seen elsewhere – for instance in decoupling the 3-particle correlation function equations describing a strongly interacting gas, or liquid [20].

**Exponential Expansion of Perturbative Series (Paper)**

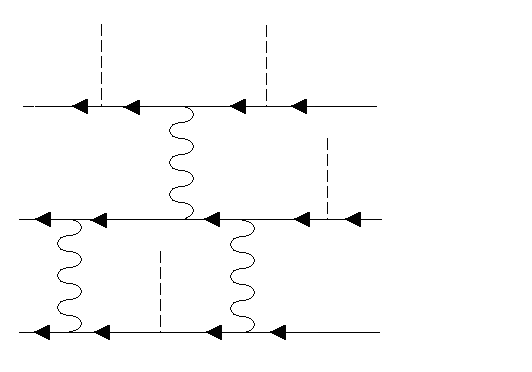
Here I’ll do it a little differently and instead of getting our final result in the two step process above, just do it all at once. Each diagram in the expansion of GN can be labeled according to the number of single particle potential lines on the each particle, and the number of interaction lines between each pair of particles. For convenience, we will then label each diagram as,



The superscripts, vi, label the number of potential lines on each particle, i = 1…N. And the subscripts, ui, label the number of potential lines on each pair of particles j = 1…N(N-1)/2. So for instance the diagram below,



would be labeled W211102 since it has 2 v’s on particle 1, 1 v on particle 2, and 1 v on particle 3, as well as 1 u between the particle pair {12}, 0 u’s between {13}, and 2 u’s between {23}. Other distinct diagrams, obtained by permuting the arrangements (times) of potential lines on this diagram would also be of the same order, W211102. For example,



which is different from the first by interchange of the first two external potential and interaction lines. So then the Taylor series expansion of the N-particle Green’s function would be…



where GN(0) is the unperturbed N-particle Green’s function, and we’ll remember that each Wν1,ν2,…,νNu1,u2,…,uN(N-1)/2 stands for a particular *class* of diagrams, with the specified number of external potential lines per particle, and interactions per pair. The different diagrams within the class can be obtained by horizontally transposing the order of the interactions. Now divide through by GN(0) to get,



And then we’d like to re-express this Taylor series as an exponential series. So we write,



To define the *f* ’s with respect to the *w*’s, we equate both expressions,



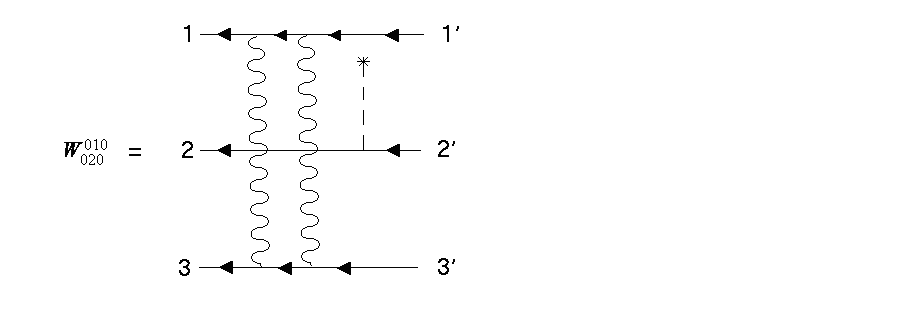
and expand the ln ‘order by order’, equating terms of order λν1,ν2,…,νNu1,u2,…,uN(N-1)/2 on both sides. This process in general defines the *f* ’s in the exponent. Formal rules can be worked out if one continues to expand the ln and equate order by order. And they are these. The N-particle Green’s function is:



There is a one to one correspondance between a particular diagram, *w*, in the Taylor series expansion of GN, and a particular term *f* in the exponential series expansion of G.



To illustrate, consider this particular diagram in the Taylor series expansion (for N = 3).



According to the rules above, the term entering into the exponential series expansion corresponding to this diagram would be:



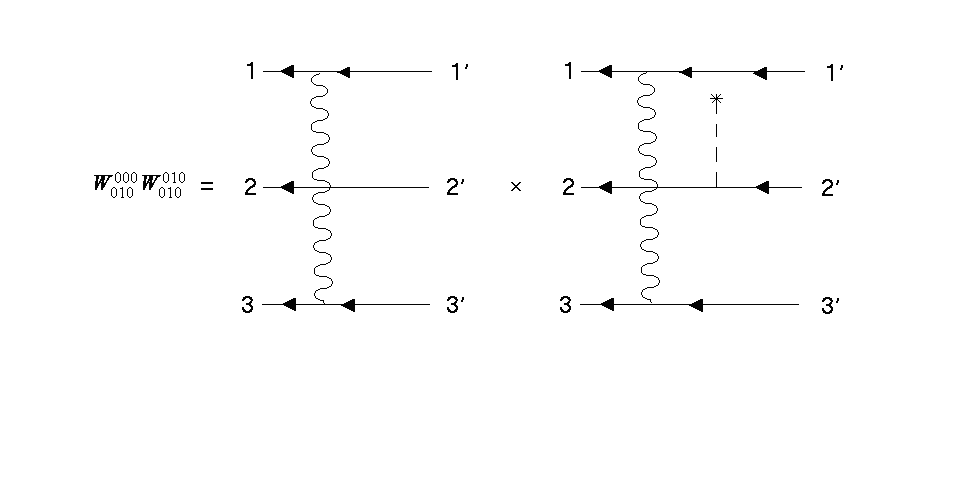
We can see this as follows. Step one tells us to start with our given dimensionless *w*, i.e.,



and this would enter intovia:



since each w carries a minus sign. Step two instructs us to cut the diagram vertically in all possible ways. So starting left to right, we first cut the diagram inbetween the two interaction lines, resulting in the product



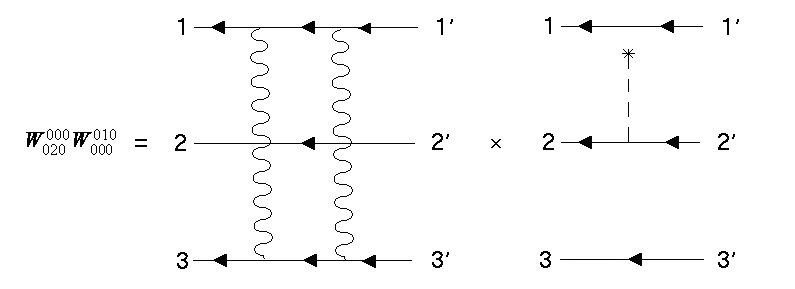
This daughter diagram would enter into f as:



Since each w carries a (-) the overall term is positive, and since there are two w’s we divide by 2. So we have so far,



Next we could cut between the {13} interaction line and external potential line, resulting in the product,



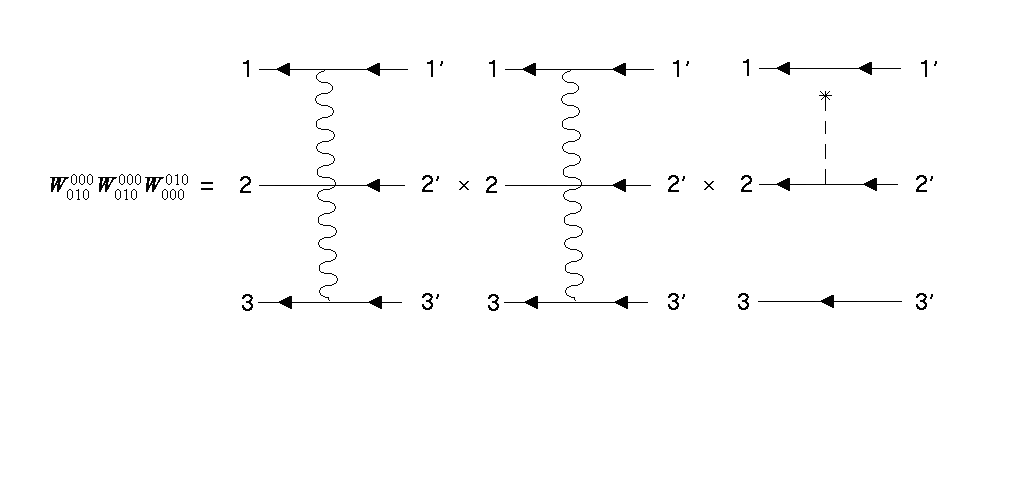
which would enter into *f* as:



giving us so far:



Finally we could cut our diagram into three pieces – along each of the interaction/external potential lines. Then we’d have the product:



which would enter into *f* as:



Since there are 3 *w*’s we divide by 3. And there is an overall (-) sign due to the odd number of *w*’s. This is all the ways we can cut it so this gives us:



Since we can cut/not cut between every interaction line, it seems overall there should be 2N-1 different ways to the cut the diagrams

**Factoring out G1’s and G2’s from the expansion**

Now we’d like to factor out of the exponential expansion the diagrams which give us G1 and G2. So going back to:



Consider the set of diagrams parameterized by: fv1,0,0,…0,0,0,…. This set has v external potential lines on particle 1, and nothing else on the other N – 1 lines. If we add these diagrams up, then we get,



Similarly adding up f0,v2,0,…0,0,0,… would give us:



Likewise, consider the set of diagrams given by: f0,0,0…u1,0,0,…. This would correspond to the set of interaction ladders between the first pair of particles, say 1 and 2, and no interaction lines/potential lines between any of the other particles. If we add these up, then we’d get,



and similarly for f0,0,0…0,u2,0,…, etc.

**Final Result**

Therefore let us separate the *f*-terms with only one non-zero index from the rest, denoted by

*f ′*.



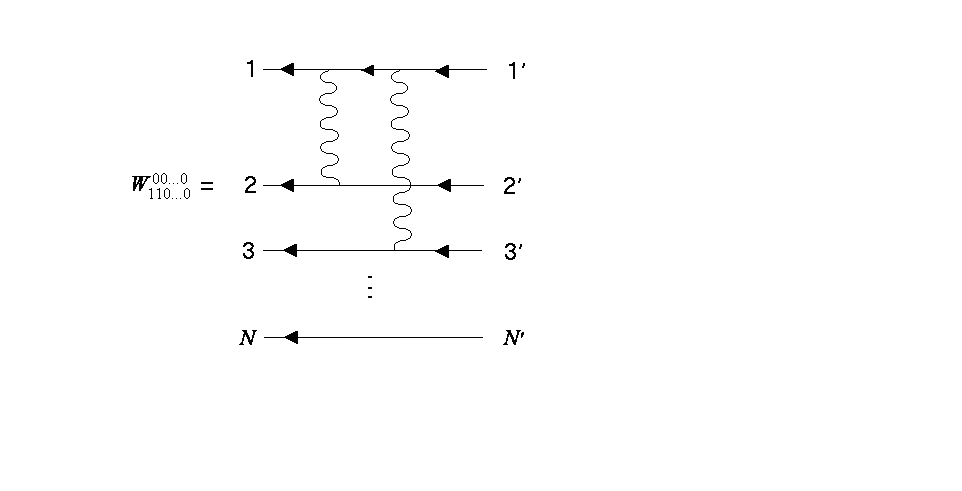
And so we have our final result,



where the *f* ′ refers to all diagrams which don’t consist of purely external potential lines on *one* particle, or interaction lines between *one* pair of particles – basically any f with more than two non-zero indices. If we neglect all fʹ diagrams, then our result will technically be accurate only up to first order in V and U…but in the same way that including only one self-energy diagram is exact only up to first order. Also note that the ‘zeroth order’ part of this expression sums all factorizable diagrams, and approximates all non-factorizable diagrams as there factorizable counterparts (see diagrammatic illustrations in Dissertation section). This is evidenced by the fact that all factorizable diagrams have 0 as their f counterpart. The fʹ diagrams correct this approximation.

**Three-particle correlations**

If we want to include 3-particle correlations, then to leading order they’d come from,



which corresponds to an *f* of:



Including all these terms, we’ have,



where we’re summing over all permutations of the 11 throughout the indices – such that the potential lines are connected to each other, since disconnected lines give an *f* of 0 [or they just cancel out with the denominator, right?]. There is only one type of diagram to consider – the one above, which we’ll call f(1,2,3). And note that we can switch the order of the potentials. So let f be the symmetrized version. The question is then, how many of these pairs are there? And over what particle pairs? Let’s consider the case of N = 3. Then we have,

{12,13} {12,23}

{13,23}

These would be represented by f(1,2,3), f(2,3,1), and f(3,1,2) respectively. In other words, this would just be a cyclic permutation of the coordinates. Let’s call F3(1,2,3) the sum of all cyclic permutations of f(1,2,3). Then the 3-particle correction term would simply be:



So we would then just sum over all distinct triplets. And I think we could write this as:



G2 would be exact to first order in the interaction. F3 would be 2nd order in the interaction (assuming no external potentials).

**Is there a way to incorporate boundary conditions on G2?**

Boundary conditions can be included in G1. I’m not sure how important they are to G2. At one level, if the 2-body Hamiltonian (the simplest N-particle problem) can be solved, then the Green’s function can be constructed from the eigenfunctions. And any situation that has already been solved for N-particles (including boundary conditions) has been solved for 2 of course. And since the eigenfunctions probably aren’t too complicated – being only 2-dimensional, the normalization, and summation can probably be accomplished. So we can go from any solved 2-body problem to the N-body problem.

**Symmetrization**

Suppose we have solution:



which satisfies the usual δ function initial conditions. If we need to symmetrize the green’s function, then we would just add permutations over the x′ coordinates. Consider just an unperturbed GF first:



If we’re interested in Z, then when we take y1 → x1 and y2 → x2 we get:



If we’re interested in G2(x), then we take y1, y2 → 0. Then we get:



which gives 0 of course. In that limit there is no distinction between y’s.