**Relating Sscatter to Stime development**

**1D S-matrix**

We might notice that the T-matrix appearing in Sscatter is the same as appears in TDPT in the context of the time-development S-matrix, STD. And so it stands to reason we can relate these two. Consider STD for the adiabatic perturbation,



and evaluate it between two free states (normalized to delta function):



But since |k> = √(k/2πm)φk(x), this is none other than,



Now recall from the RSPT scattering 1D file, the result we found for the scattering S-matrix (boxed between two φm,nstates):



And so we have:



Yay! So our time-development matrix is just Sscatter restricted to the constant energy surface of phase space). That kind of makes sense, doesn’t it. Might note that this simplifies rather nicely to:



Moving on…

**Q1D**

And we’d presume, like in 1D, that we can relate the scattering S matrix to the time-development S-matrix via:



**3D**

And we’d like to relate the 3D Sscattering matrix to STD, just as with the others. Let’s evaluate the latter between two normalized 3D free states:



So we need the relationship between δ(**k**´-**k**) and δ(Ek´ - Ek). So let’s say,



and try to figure out the constant of proportionality. We must have of course that the integral of LHS over d3k equals 1…I’m going to set ℏ = 1, ‘cause it’s not coming out right and so I’ve misplaced a factor of it somewhere but who cares.



So,



Now recall from the RSPT scattering file in 3D the result (in that file we didn’t set ℏ = 1):



So like with the 1D case, we find the S-matrix is just STD restricted to the constant energy surface!



**Levinson’s Theorem and all that**

So it was alleged that the phase shift, δ±(k), or δℓ(k), which is related to the (scattering) S-matrix, S±(k), or Sℓ(k), via S = e2iδ, contains within it the information about the bound/metastable, etc., states of the system via analytic continuation of variable k to a complex variable. Basically, the poles of δ, and hence Sscatter, were the bound/metastable states. This now makes sense because we know that:



And so the poles of Sscatter (Ek analytically continued) will be the poles of T (Ek analytically continued). But T (Ek analytically continued) is just:



and we know from the constant TD perturbation file, that the GF is:



and we established that the poles of G were indeed the bound/metastable/scattering (branch cuts) states.