**Charge in a magnetic field**

Now let’s look at the same problem, a charge in a magnetic field **B** = B**k**, using a different gauge: the Landau gauge **A** = (0, Bx, 0)**.**  Can verify that **B** = ∇×**A** still. Note that physically, and this holds for the last file too, this B would be the bulk-interstitial field generated by external fields, as well as the fields generated by other particles internal to the system. Either way, the field is certainly external to our single atom. The Landau gauge has certain advantages over the symmetric gauge – seems that it’s generally easier to calculate stuff in it. So anyway, jumping in, the Hamiltonian is:



(e here includes sign of charge: negative for electrons, positive for, uh, positrons) We try a wavefunction of the form,



and we end up with the Schrödinger equation,



and so,



which is the harmonic oscillator equation. And so the eigenfunctions/eigenvalues are fairly obviously,



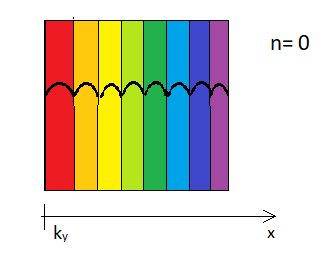
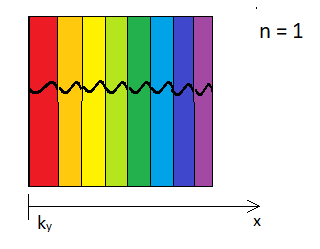
And will recall that the first few Hermite polynomials (see Harmonic Oscillator) are:



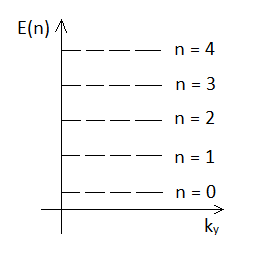
So in this gauge we get plane waves in the y-direction, and Gaussian’s in the x (kind of makes sense, looking at **A**). Note there is little connection between ky and velocity vy (because B disrupts the correspondence v and k). In fact <vy> = 0 (see below).

Still, the larger ky is, the further the x center of the wavefunction is shifted horizontally, which would make sense from a magnetic force point of view if we were able to interpret ky as some kind of vy. Also, note that the width of the wavefunction in the x-direction, ℓB, is quite quite small.

Interestingly the energy levels depend only on n (the Landau level) which measures x-kinetic/magnetic energy (represented by curvature of the black line) and are independent of ky so we see the degeneracy popping up again. An author notes that we could take these degenerate states, and form them into linear combinations that would match the form of the degenerate states we saw in the symmetric gauge. Anyway, we can represent these wavefunctions like this:

etc. Each colored rectangle represents a given wavefunction. The color is the value of ky. The black line is the shape of the wavefunction in the x-direction: n = 0 corresponds to Gaussian hump, and n = 1 is whatever that HO wavefunction is, etc….in reality the wavefunctions / rectangles would overlap considerably, and in fact get wider (so more overlap) as n increases. Might make an energy level diagram like this (ignoring the kz part of the energy spectrum), noting the degeneracy along the ky quantum number:



**Degeneracy of energy levels**

We can work out the degeneracy of the energy levels if we impose boundary conditions on our wavefunction so that it is restricted to a rectangular space with length Lx, and width Ly. So our wavefunction is:



and we’ll require that the wavefunction be periodic in the y-direction (or could say 0 at the boundaries in the y-direction, but this amounts to the same, and is easier to work with). This requires, where m is an integer,



and therefore quantizes ky. Next we require that the center of the wavefunction(s) fit within the sample (in the x-direction). So the center is:



So the higher the (quantized) ky is, the further the wavefunction’s center is from x = 0. Note its position doesn’t depend on n. The spacing between centers is obviously,



Now the degeneracy of each level is obtained by calculating the number of centers that will fit between 0 and Lx. This is:



So we have:



From the diagram above, we might have thought the degeneracy to be Lx/ℓB, but must keep in mind the appreciable overlap between wavefunctions which the diagram doesn’t convey.

Note that the free problem (B = 0) would just be a bunch of the typical plane waves. And the lowest lying states would have practically zero degeneracy. This is reflected in fact that as B 🡪 0, the centers of the strips spread further apart. But interestingly, as B increases the degeneracy does too. Spacing between levels is proportional to B, as is degeneracy.

What is the degeneracy, roughly, for a 2D substance, say? Well,



where N os the number of particles. So for fields ranging from B = 0T → 10T, we’ll go from N = roughly 0 (or 1) particles → 1000 particles per Landau level. And the number of Landau levels occupied will go from roughly N to N/1000. It’s definitely a lot either way.

**Flux Quantization?**

So our wavefunction is:



Could write the ψn(HO)’s as:



To get the flux penetrating a given wavefunction, we need to know its width. We’ll take this to be the distance between the two furthest local max’s as the width. Now recall,



So Hn(ξ) ~ 2nξn. And so need to find peak of:



This is at:



And the corresponding peak in the -x direction will be at -√n. So,



So the flux going through our wavefunction would be:



Unlike what we found with the symmetric gauge, this is clearly not independent of the sample dimensions and magnetic field strength. So it seems that, yeah, flux quantization through a wavefunction isn’t really a thing.

**Some Expectations**

For instance, consider the velocity expectation of these states [note **v** = **p** – e**A**]:



Now let’s consider the expectation of the *physical* angular momentum in the z-direction. This is:



To get <x2> we’ll do something like (don’t need y or z part of wavefunction)



We can do that integral, as it’s just the expectation of PE = (1/2)mω2x2 divided by (1/2)mω2. And expectation of PE is half the expectation of the energy (since KE and PE are distributed equally in a HO). Of course the expectation of the energy is ωc(n+1/2). So,



Therefore we have:



and so,



That’s a rather simple result. But it makes sense. Consider the magnetic moment. This would be **M** = γ**L**phys (γ = e/2m – see EM folder/asymptotic expansion). And so,



Note this is a purely diamagnetic response to the magnetic field, as we were hoping for in the last file. But we also know that we should just have <Mz> = -∂<E>/∂B, which would imply,



So off by a factor of 2? Hmmm….Let’s check another way. Let’s factor all the B-stuff out of H:



So,



Comparing with:



We see that we clearly do *not* have these guys matching, unlike was the case in the symmetric gauge. Well, probably **M** = γ**L**phys doesn’t hold true in all gauges? Maybe I’m missing something? I’m pretty sure **M** = -∂<E>/∂**B** is always true though, as it kind of follows from thermodynamics [see Thermodynamics/Equilibrium Systems, and make note that the B in our present context would be equivalent to Bf in that file’s context, as our present B is the *external* field].

**Adding Spin**

If we were to add spin to this problem, then we’d have:



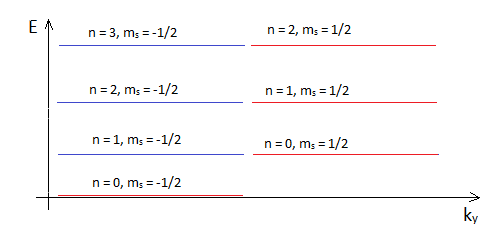
where **μ** = gγ**S** (and γ = e/2m), and the wavefunctions, energies are same as before, with the addition of a spinor. And we’ll have:



For electrons this simplifies to:



The energies for electrons will look the same as before except that each level will split into two (and ignoring the kz part of the energy spectrum): the lower for the magnetic moment up (spin down), and higher for the magnetic moment down (spin up), and again ignoring the kz part of the spectrum:



And each level (blue or red line) has degeneracy N = Φ/Φ0. I’ll add the comment that now we have:



which for electrons is of course,



Finally, should see that there are two responses here: a diamagnetic response coming from the orbital motion of the electron, and a paramagnetic response coming from the spin.