**Rotation Operator**

Just gonna do a bunch of examples,

**Example**

Consider the following Hamiltonian, and check to see if its invariant under rotations.



Calculating,



because  and  are the magnitudes of the respective vector (operators), and rotating a vector doesn’t change its magnitude.

**Example**

Calculate to first order the effect of D(α**k**) on |x**,**y>. So we have:



The action of this operator may be inferred with the recognition that to first order in **d**, we may replace,



and so this is, assuming α is very small…



**Example**

Show explicitly what we get by rotating the , 90° about the z-axis, and calculate similarly the effect on the .

We expect to get - and respectively, given our discussion of R(**α**) in the previous file. One approach is to use the differential equation. But for that we have to consider general rotations. So consider an arbitrary rotation about the z-axis:



Then



obeys the differential equation:



To proceed we need to now evaluate how the y-operator changes. So consider,



Now we have two coupled ODE’s. We can solve them for x(α). To proceed, differentiate the first and plug in the second,



The solution is of course,



where A and B are undetermined operators. But we can determine them. We know that when α = 0, this must reduce to x. And so A = x. Furthermore, the derivative is -y, and so we must have:



and when α = 0 this should reduce to y. So we must have B = -y. Therefore we have:



We can write this as:



which is,



which is exactly the form we were to expect. So when we have an α = π/2 rotation we get,



as expected. And this makes sense, ‘cause then,



and are the initial components of the vector. (α), (α) are the components of the rotated vector, in terms of the original components. So when we get x(π/2) = -y, this is in accordance with the diagram since y was zero. And y(π/2) = x makes sense because the y-component of our rotated vector is indeed equal to the initial vector’s x-component.

**Example**

Show that D(π/2**j**)|ℓm> (where ℓ = 1 and |ℓm> are the Lz eigenkets) gives the Lx eigenkets, as asserted in previous file. So we have:

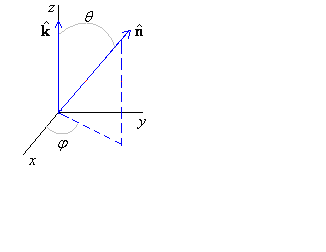


precisely as found earlier (well, see Time-Independent/Angular momentum file).

**Example**

What are the simultaneous eigenkets of the operators L2 and Ln = **L**·**n** where **n** is a unit vector pointing in some direction (for ℓ = 1).

First note that we already know what they are for **n** = **k**, and **n** = **i** (which we determined above). We can get the general eigenkets from the rotation operator. We just rotate the Lz eigenkets into the **n** direction. So consider an **n** described by the angles,



The rotation can be accomplished by rotating **k** by an angle θ about the y-axis, and then an angle φ about the z-axis. In other words we have:



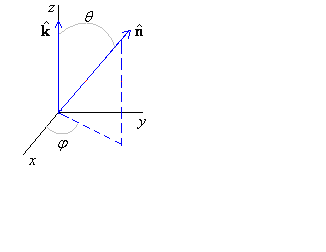
So we explicitly have for (ℓ=1)



**Example**

What are the simultaneous eigenkets of the operators S2 and Sn = **S**·**n** where **n** is a unit vector pointing in some direction (for ℓ = 1).

First note that we already know what they are for **n** = **k**, and **n** = **i** (which we determined above). We can get the general eigenkets from the rotation operator. We just rotate the Sz eigenkets into the **n** direction. So consider an **n** described by the angles,



The rotation can be accomplished by rotating **k** by an angle θ about the y-axis, and then an angle φ about the z-axis. In other words we have:



So we explicitly have for (s=1/2)



**Example. Explicit spin rotation operator**

Let’s work out the rotation operator using a few properties of Pauli matrices.

First, consider the following construction,



So,



And consequently



or for short, leaving out the implicit identity matrix,



Note it follows in particular that:



With that established, consider:



So,

