**Path Integral Formulation of GF’s**

Say we have a Hamiltonian:



Let’s consider evaluating correlations like this:



where xH(t) is the Heisenberg developed operator – developing according to the full H, and|GS> is the true ground state of the interacting problem. Can this be converted to a path integral? The first thing we need to do is to put the |GS> in terms of an eigenstate of the operator , i.e., |xa>, where xa is some coordinate. This can be done via the mass-gap theorem. Consider the time development of |xa>, and <xb|. We have variously,



where |n> are the eigenstates of the interacting problem. Now in the large-time limit, if the energy levels En are separated, only the lowest energy level term should matter, as the others would wash out in comparison (since their frequency of oscillation is much greater). So we can approximately say:



(taking t­a,b → ∞ limit implicitly) Solving for |GS>, we have:



And then have:



Now we can get rid of |GS> by recognizing, from above, that:



so,



and inserting the U’s implicit within (t), we have:



where TC orders the operators from 0 to t´ to t to 0. We can now remove the TC operator [need to be more clear on this point], and write:



Now insert resolutions of identity:



and the denominator may be written as a path integral too, giving us,



Now we take the ta,b → ∞ limit explicitly. And when we do, it seems the boundary values disappear. Perhaps that is because in this limit, the starting/ending points have negligible contribution. So we have:



The Feynman rules derived from this path integral just replicate the ones already discussed. And we’ll note the denominator will cancel the vacuum bubbles in the numerator. But we may still have disconnected diagrams where x(t) and x(t´) are not connected to each other.