**Self-Energy**

Let’s do a quick example,

**Example of HO**

Let’s consider



What is the self-energy to first order in this interaction?



So,



(factor of 1/2 comes from coincident propagator) and so,



This is entirely real. So our GF would be:



(had to add the iη back in – just kind of figured I could neglect it b/c ΣC would have imaginary part, but it doesn’t in this case) So the new excitation is:



In particular, we’d now estimate the energies within this model to be:



The E0 is there because we can only estimate the excitations via this method, not the ground state energy. To go beyond perturbation theory (i.e., basically assuming the interaction, u, is small), we’d have to sum an infinite number of diagrams. Obviously we cannot sum them all. But we can sum a subset. One popular way to do this is to approximate the self-energy self-consistently using the first self-energy diagram:



The bold line is the exact GF to within this approximation. So we’d have:



where,



(not putting argument on ΣC to emphasize it doesn’t depend on ω) Filling the top equation into the bottom gives us:



Now we can solve for ΣC…



This is a cubic equation which *can* be solved exactly but I won’t bother. Will note that in limit of weak coupling, u → 0, and then ΣC should too. In that case our equation reduces to:



which is what we found above. In the limit of strong coupling, presumably ΣC will grow large, and we’d have:



Here we can see that ΣC grows as u2/3, not even to an integer power. And so obviously, every single term in our naïve perturbative series would’ve been untrustworthy if taken to the large u limit. Filling back into our GF,



we see the new excitation would be something like,



and new energies,



(in the high interaction strength limit. Of course, even our result here is not necessarily trustworthy, as we’d have to make a successful case for why a self-consistent approximation based off of the very first self-energy diagram should make any sense. But this at least gives us an idea of how to go beyond perturbation theory. Of course we *could* (somehow) exactly solve the Schrodinger equation in this case, and it’d be interesting to compare the exact results with our approximations.