**RSTPT Perturbation Theory**

**Perturbative Expansion**

As was the case with the regular Schrodinger equation, 99% of the time, we cannot actually solve the time-dependent Schrodinger equation exactly. For example, the solution to the free particle time-dependent Schrodinger equation we managed to solve, but if we add even a constant force to the equation (note V(x) = -Fx if F is constant)



Then it becomes pretty hard to solve the equation explicitly in closed form. Clearly quantum dynamics is a lot harder than classical dynamics. So we have to resort to some sort of perturbation theory approach again. There are two kinds that figure prominently, the Raleigh-Schrodinger perturbation theory, which is probably the most common, and a path-integral perturbation theory, developed by Richard Feynman (as a graduate student – smart people suck!), which we’ll explore via the WKB approximation. But let’s start with the RSTDPT. So we can calculate the transition probability from one unperturbed state to another when subject to such a perturbation, V(t). This is:



And we’ll recall,



Solving for Sni(t,t0) perturbatively…Filling in interaction picture operators, and resolutions of identity, we have:



which is (just keeping up to the third order term for concision, and implicit summation over repeated indices):



We can write this in a cleaner way. We’ll start by introducing theta functions.



The t´ integral can be extended to -∞ because it’s restricted to be greater than t´´, t´´´ in the second, third order terms, which themselves are restricted to be greater than t0. Now we’ll define the Green’s Function (GF). It’s defined as:



And we can write a pretty expression for the series expansion of this guy. So consider Gni(t,t0) = <n|(t,t0)|i>.



Then the GF is …



which is:



(we can get rid of the θ(t-t0) terms next to the integrals because, e.g. in the first order term, t is already restricted to be greater than t´, which is restricted to be greater than t0, and so t is already greater than t0 automatically) which is:



And finally,



where we define:



which we’ll note is just:



We usually just keep the first order term in the expansion. Note that second order processes involve transitions from an initial state to an intermediate one, to a final one. The intermediate transitions don’t have to conserve energy (so called virtual transitions) but the overall transition does. Anyway, now we see,



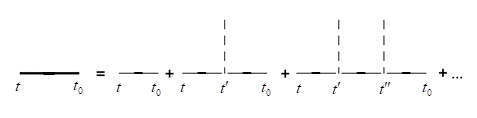
where we have *implicit summation over repeated indices*, and where recall,



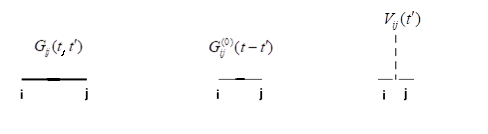
Relating this back to S, we can write:



We can represent G diagrammatically as:



where,



In terms of the ‘operator’ GF, this series can be written even more concisely,



where,



and relating everything back to S, we have:



The burning question is, ’is it acceptable to keep just a few terms in the series?’. The answer would seem to be a much qualified yes. As long as each successive term is smaller than the previous then we can. This will usually require t-t0 be super small, since the exponent of S, i.e., ∫t0t VI(t´)dt´, would seem to grow with t-t0. Seems we typically need t << 1/|En – Ei|. Secondly, we’ll see that we can typically trust the truncated series for Gni only as long as Vni << |En – Ei|, and more generally, for higher order terms, that Vab << |Ea – Eb|. We might note that this utterly fails for transitions between the same state as we can’t get smaller than 0. So basically – only for small t and small V.



This is also as we saw in our spin examples.